**Redshifts and Distances**

Spacetime and Geometry : An Introduction to General Relativity – by Sean M Carroll

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# Summary

In section 8.5 we are looking at redshifts and distances. The latter are more complicated than you might think! We start in an FLRW universe with metric

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| --- | --- |
|  | (1) |

We then found a rank 2 Killing tensor

|  |  |
| --- | --- |
|  | (2) |

where is the 4-velocity of (all) comoving observers and . We proved it is a Killing tensor in 'Commentary 8.5 Killing Tensor in FLRW spacetime'.

Summarising the business:

* We use to show that interstellar gas cools in an expanding universe (7).
* Then we use it to show that as the universe expands the observed frequency of a photon will decrease (16).
* Cosmologist measure the redshift, , of distant objects and we show how this tells us the scale factor when the object emitted the light being measured (20).
* I then looked up some galactic distances and redshifts to compare them. Extrapolating the graph indicates the size of the visible universe is ~15 Gly (21).
* Carroll then seems to ramble a bit and goes back to a non-relativistic redshift (36). The history of Doppler and Hubble, which he does not mention, is quite interesting.
* Getting back on track Carrol shows that the nearby universe can be thought of as flat and we are able to use proper distance (42) meaningfully and derive Hubble's law (46). I think he is overcomplicated.
* Next step other distances!

# The Business

Extending our rule about Killing vectors a bit (Carroll 3.175) we have is conserved along geodesics. is the four-momentum of some (any) particle. Mass is conserved so that means if its four velocity is

|  |  |
| --- | --- |
|  | (3) |

is conserved for the particle along geodesics. Call the conserved quantity and

|  |  |
| --- | --- |
|  | (4) |

is constant for the particle along geodesics.

## Massive particles (interstellar gas)

For massive particles and so

|  |  |
| --- | --- |
|  | (5) |

where we introduced where is very like the three-velocity of the particle. Since we are thinking of a particle on a geodesic, it is in free-fall. We also have so (4) becomes

|  |  |
| --- | --- |
|  | (6) |
|  | (7) |

which is Carroll's 8.101 and shows that particles in free fall "slow down" as the scale factor increases. I suppose the quotes are there because is not quite your usual three velocity which is but the quantity . Also we have not used the true fact that or Carroll's equation 8.100 which was also true. Both seem irrelevant.

As Carroll says free moving particles, such as interstellar gas, slow down. or cool, as the universe expands. Is that surprising?

## Photons

For photons the line element equation is

|  |  |
| --- | --- |
|  | (8) |
|  | (9) |

where is the four-velocity of the photon. So that's the easy place where Carroll's comes from.

Stick that into (4) and we get

|  |  |
| --- | --- |
|  | (10) |
|  | (11) |

Then back in section 3.4 we had equations 3.62 and 3.63: The four-momentum of a photon is

|  |  |
| --- | --- |
|  | (12) |

Any observer with four-velocity measures energy of a photon with four momentum to be

|  |  |
| --- | --- |
|  | (13) |

and we proved that in 'Commentary 3.4 Particle energy'. Of course we have

<https://en.wikipedia.org/wiki/Edwin_Hubble>